Quantum computation for beginners.

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Presentation overview

- Basics concepts of quantum computation
- Available quantum hardwares and softwares

Quantum Computation

The use of quantum phenomenons to compute.

- Superposition
- Entanglement

Quantum Computation

The use of quantum phenomenons to compute.

- Superposition
- Entanglement

Solve **some problems** faster the a classical computer.

- Integer factorization - Shor's algorithm (a NP problem in polynomial time)
- Unordered database search - Grover's algorithm (a $O(n)$ problem in $O(\sqrt{n})$)

Quantum bit

Qubit

Bit	Qubit
0 or 	$\vert 0\rangle$ or $\vert$$\rangle$
1 or 	$\vert 1\rangle$ or $\vert$$\rangle$

Quantum bit

Qubit

Bit

- 0 or 
- 1 or 

Qubit

- $|0\rangle$ or $|$$\rangle$
- $|1\rangle$ or $|$$\rangle$
- $|0\rangle + |1\rangle$ or $|$$\rangle + |$$\rangle$
- $|$$\rangle + |$$\rangle$

Superposition

Classical

- `char[4] name = "Gugu";`

Quantum

- `quantum char[4] name = "Dedé" + "Didi" + "Guga" + "Gugu" + "Pelé" + "Xuxa" + ···;`

Superposition

Classical

- `char[4] name = "Gugu";`
- 3 bits: 0, 1, 2, ..., 7

Quantum

- `quantum char[4] name = "Dedé" + "Didi" + "Guga" + "Gugu" + "Pelé" + "Xuxa" + ···;`
- 3 qubits: $|0\rangle + |1\rangle + |2\rangle + \cdots + |7\rangle$

Superposition

Classical	Quantum
▶ `char[4] name = "Gugu";`	▶ `quantum char[4] name = "Dedé" + "Didi" + "Guga" + "Gugu" + "Pelé" + "Xuxa" + ⋯;`
▶ 3 bits: 0, 1, 2, ..., 7	▶ 3 qubits: $\|0\rangle + \|1\rangle + \|2\rangle + \cdots + \|7\rangle$

N qubits $\equiv 2^N$ bits

Superposition

5 qubits in superposition

$$\frac{1}{\sqrt{2^5}}\sum_{k=0}^{31}|k\rangle = \frac{1}{\sqrt{32}}\begin{pmatrix} |00000\rangle + |00001\rangle + |00010\rangle + |00011\rangle + \\ |00100\rangle + |00101\rangle + |00110\rangle + |00111\rangle + \\ |01000\rangle + |01001\rangle + |01010\rangle + |01011\rangle + \\ |01100\rangle + |01101\rangle + |01110\rangle + |01111\rangle + \\ |10000\rangle + |10001\rangle + |10010\rangle + |10011\rangle + \\ |10100\rangle + |10101\rangle + |10110\rangle + |10111\rangle + \\ |11000\rangle + |11001\rangle + |11010\rangle + |11011\rangle + \\ |11100\rangle + |11101\rangle + |11110\rangle + |11111\rangle \end{pmatrix}$$

Superposition

6 qubits in superposition

$$\frac{1}{\sqrt{64}}\sum_{k=0}^{63}|k\rangle = \frac{1}{\sqrt{2^6}}\begin{pmatrix}
|000000\rangle + |000001\rangle + |000010\rangle + |000011\rangle + \\
|000100\rangle + |000101\rangle + |000110\rangle + |000111\rangle + \\
|001000\rangle + |001001\rangle + |001010\rangle + |001011\rangle + \\
|001100\rangle + |001101\rangle + |001110\rangle + |001111\rangle + \\
|010000\rangle + |010001\rangle + |010010\rangle + |010011\rangle + \\
|010100\rangle + |010101\rangle + |010110\rangle + |010111\rangle + \\
|011000\rangle + |011001\rangle + |011010\rangle + |011011\rangle + \\
|011100\rangle + |011101\rangle + |011110\rangle + |011111\rangle + \\
|100000\rangle + |100001\rangle + |100010\rangle + |100011\rangle + \\
|100100\rangle + |100101\rangle + |100110\rangle + |100111\rangle + \\
|101000\rangle + |101001\rangle + |101010\rangle + |101011\rangle + \\
|101100\rangle + |101101\rangle + |101110\rangle + |101111\rangle + \\
|110000\rangle + |110001\rangle + |110010\rangle + |110011\rangle + \\
|110100\rangle + |110101\rangle + |110110\rangle + |110111\rangle + \\
|111000\rangle + |111001\rangle + |111010\rangle + |111011\rangle + \\
|111100\rangle + |111101\rangle + |111110\rangle + |111111\rangle
\end{pmatrix}$$

Superposition

1 qubit

- $\alpha |0\rangle + \beta |1\rangle$
- $\alpha, \beta \in \mathbb{C}$
- $|\alpha|^2 + |\beta|^2 = 1$

N qubits

- $\sum_{k=0}^{2^N-1} \alpha_k |k\rangle$
- $\alpha_k \in \mathbb{C}$
- $\sum_{k=0}^{2^N-1} |\alpha_k|^2 = 1$

Complex number ($\mathbb{C}$)

- $i = \sqrt{-1}$
- $z = a + bi$
- $z = re^{i\varphi} = r(\cos\varphi + i\sin\varphi)$

Measurement

1. $(|2\rangle + |4\rangle + |6\rangle)$

Measurement

1.  $(|2\rangle + |4\rangle + |6\rangle) \to 2$

Measurement

1. 👁 $(|2\rangle + |4\rangle + |6\rangle) \to 2$
2. $|2\rangle$

Measurement

1. $👁 \left(|2\rangle + |4\rangle + |6\rangle\right) \to 2$
2. $|2\rangle$

1. $👁 \left(\alpha\,|0\rangle + \beta\,|1\rangle\right) \to \begin{cases} 0, & p = |\alpha|^2 \\ 1, & p = |\beta|^2 \end{cases}$
2. $\begin{cases} \dfrac{\alpha}{|\alpha|}\,|0\rangle\,, & \text{for } 👁 = 0 \\ \dfrac{\beta}{|\beta|}\,|1\rangle\,, & \text{for } 👁 = 1 \end{cases}$

Measurement

- $\text{👁}\left(\sum_{k=0}^{2^N-1} \alpha_k \left|k\right\rangle\right) \to k,\ \ p = |\alpha_k|^2$

1. $\text{👁}_0\ \left(\frac{1}{2}\left(|00\rangle + |01\rangle + |10\rangle + |11\rangle\right)\right) \to \begin{cases} 0, & p = \left|\dfrac{1}{2}\right|^2 + \left|\dfrac{1}{2}\right|^2 = 0.5 \\ 1, & p = \left|\dfrac{1}{2}\right|^2 + \left|\dfrac{1}{2}\right|^2 = 0.5 \end{cases}$

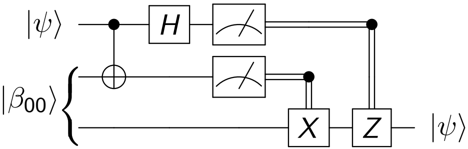
2. $\begin{cases} \dfrac{1}{\sqrt{2}}\left(|00\rangle + |01\rangle\right), & \text{for } \text{👁}_0 = 0 \\ \dfrac{1}{\sqrt{2}}\left(|10\rangle + |11\rangle\right), & \text{for } \text{👁}_0 = 1 \end{cases}$

Entanglement

1. $\text{👁}_0 \left(\frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \right) \to \begin{cases} 0, & p = \left| \frac{1}{\sqrt{2}} \right|^2 = 0.5 \\ 1, & p = \left| \frac{1}{\sqrt{2}} \right|^2 = 0.5 \end{cases}$

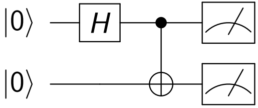
2. $\begin{cases} |00\rangle, & \text{for } \text{👁}_0 = 0 \\ |11\rangle, & \text{for } \text{👁}_0 = 1 \end{cases}$

Quantum circuit



- $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$
- $|\beta_{00}\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$

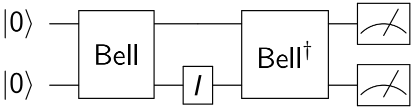
Quantum circuit



- $H\,|0\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle + |1\rangle)$
- $H\,|1\rangle = \frac{1}{\sqrt{2}}\,(|0\rangle - |1\rangle)$
- $\text{CNOT}|a\,b\rangle = |a\,(a \oplus b)\rangle$

1. $H_0\,|00\rangle = \frac{1}{\sqrt{2}}\,(|00\rangle + |10\rangle)$
2. $\text{CNOT}\frac{|00\rangle+|10\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}\,(|00\rangle + |11\rangle)$
3. Measure

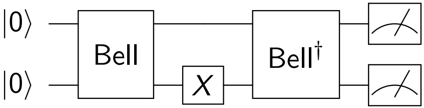
Quantum circuit



- $I\,|\psi\rangle = |\psi\rangle$

1. $\text{Bell}|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. $I_1 \frac{|00\rangle+|11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
3. $\text{Bell}^\dagger \frac{|00\rangle+|11\rangle}{\sqrt{2}} = |00\rangle$
4. Measure

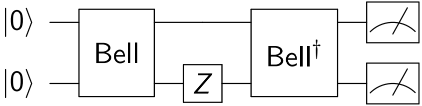
Quantum circuit



- $X|0\rangle = |1\rangle$
- $X|1\rangle = |0\rangle$

1. $\text{Bell}|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. $X_1 \frac{|00\rangle + |11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$
3. $\text{Bell}^{\dagger} \frac{|01\rangle + |10\rangle}{\sqrt{2}} = |01\rangle$
4. Measure

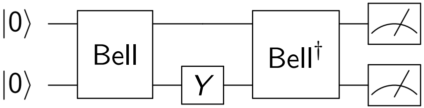
Quantum circuit



- $Z|0\rangle = |0\rangle$
- $Z|1\rangle = -|1\rangle$
- $H\frac{|0\rangle+|1\rangle}{\sqrt{2}} = |0\rangle$
- $H\frac{|0\rangle-|1\rangle}{\sqrt{2}} = |1\rangle$

1. $\text{Bell}|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. $Z_1\frac{|00\rangle+|11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
3. $\text{Bell}^\dagger\frac{|00\rangle-|11\rangle}{\sqrt{2}} = |10\rangle$
   - 3.1 $\text{CNOT}\frac{|00\rangle-|11\rangle}{\sqrt{2}} = \frac{1}{\sqrt{2}}(|00\rangle - |10\rangle)$
   - 3.2 $H_0\frac{|00\rangle-|10\rangle}{\sqrt{2}} = |10\rangle$
4. Measure

Quantum circuit



- $Y\,|0\rangle = i\,|1\rangle$
- $Y\,|1\rangle = -i\,|0\rangle$

1. $\text{Bell}|00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
2. $Y_1 \frac{|00\rangle+|11\rangle}{\sqrt{2}} = \frac{i}{\sqrt{2}}(|01\rangle - |10\rangle)$
3. $\text{Bell}^\dagger \frac{i|01\rangle - i|10\rangle}{\sqrt{2}} = i\,|11\rangle$
4. Measure

The math

Classical computation

- Boolean algebra
- AND, OR, NOT

Quantum computation

- Linear algebra
- Matrix multiplication

The math

The postulates of quantum mechanics by Michael A. Nielsen and Isaac L. Chuang

Postulate 1: Associated to any isolated physical system is a complex vector space with inner product (that is, a Hilbert space) known as the *state space* of the system, The system is completely described by its *state vector*, which is a unit vector in the system's state space.

Postulate 2: The evolution of a *closed* quantum system is described by a *unitary transformation*. That is, the state $|\psi\rangle$ of the system at time t_1 is related to the state $|\psi'\rangle$ of the system at time t_2 by a unitary operator U which depends only on the times t_1 and t_2, $U|\psi\rangle = |\psi'\rangle$.

The math

The postulates of quantum mechanics by Michael A. Nielsen and Isaac L. Chuang

Postulate 3: Quantum measurements are described by a collection $\{M_m\}$ of *measurement operators*. These are operators acting on the state space of the system being measured. The index m refers to the measurement outcomes that may occur in the experiment. If the state of the quantum system is $|\psi\rangle$ immediately before the measurement then the probability that result m occurs is given by $p(m) = \langle\psi| M_m^\dagger M_m |\psi\rangle$, and the state of the system after the measurement is $\frac{M_m|\psi\rangle}{\sqrt{p(m)}}$. The measurement operators satisfy the *completeness equation*, $\sum_m M_m^\dagger M_m = I$.

Postulate 4: The state space of a composite physical system is the tensor product of the state spaces of the component physical systems Moreover, if we have systems numbered 1 through n, and system number i, is prepared in the state $|\psi_i\rangle$, then the joint state of the total system is $|\psi_1\rangle \oplus |\psi_2\rangle \otimes \cdots \otimes |\psi_n\rangle$.

Quantum hardware and software

IBM Quantum

Quantum hardware

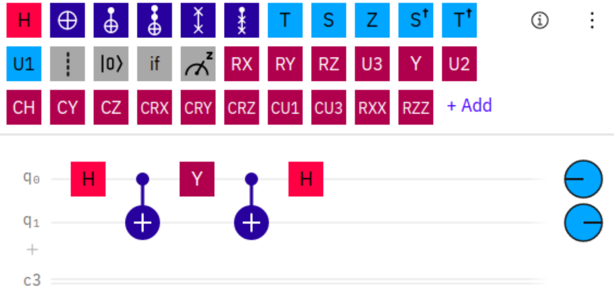
- Superconducting qubits
- -273,13 °C
- Open access
  - 1, 5, and 15 qubits
- Premium access
  - 5, 27, 28, 53, and 65 qubits

IBMQ System One.
Font: ibm.com/quantum-computing.

IBM Quantum

Software

- Free quantum simulator
  - up to 32 qubits
- Qiskit - Python
  - Terra - Circuit composer
  - Aer - Simulator
  - Ignis - Q. noise
  - Aqua - Q. algorithms
- IBM Quantum Experience

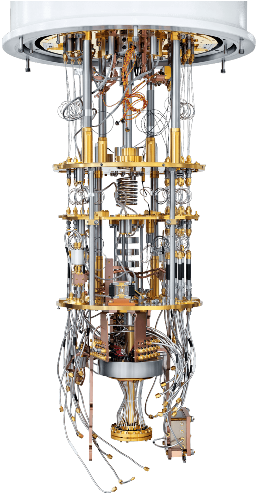


Circuit composer.
Screenshot from quantum-computing.ibm.com.

Rigetti

Quantum hardware

- Superconducting qubits
- -273,13 °C
- aws Cloud Access
  - 31 qubits



Rigetti quantum computer.
Font: rigetti.com.

Rigetti

Software

- Quil - Quantum instruction language
- Forest SDK
  - pyQuil - Python library
  - Quilc - Quil compiler
  - QVM - Quantum Virtual machine

```
from pyquil import get_qc, Program
from pyquil.gates import CNOT, H, MEASURE

qvm = get_qc('2q-qvm')

p = Program()
p += H(0)
p += CNOT(0, 1)
ro = p.declare('ro', 'BIT', 2)
p += MEASURE(0, ro[0])
p += MEASURE(1, ro[1])
p.wrap_in_numshots_loop(10)

qvm.run(p).tolist()
```

pyQuil example.

IonQ

Quantum hardware

- Trapped ion quantum computer
- Room temperature
- aws Cloud Access
  - 11 qubits
  - in late-2021: 32 qubits

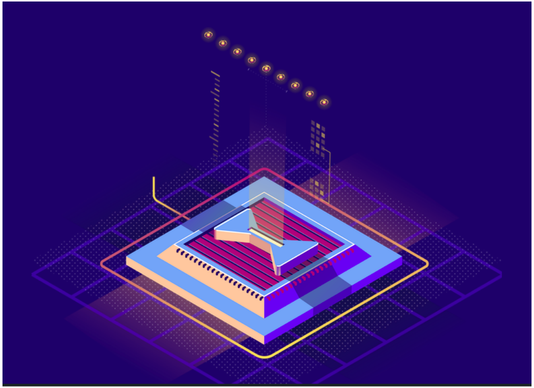


Illustration of a trapped ion quantum computer.
Screenshot from ionq.com/technology.

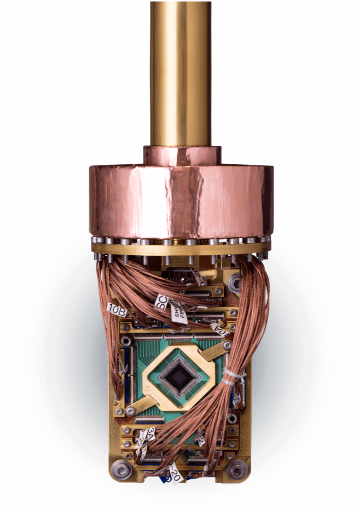
D-Wave

Quantum hardware

- Superconducting qubits
- -273,13 °C
- Quantum annealing
- Free limited access
- aws Cloud Access
  - D-Wave 2000Q : 2000 qubits*
  - D-Wave Advantage: 5000 qubits*

Software

- Ocean - Python



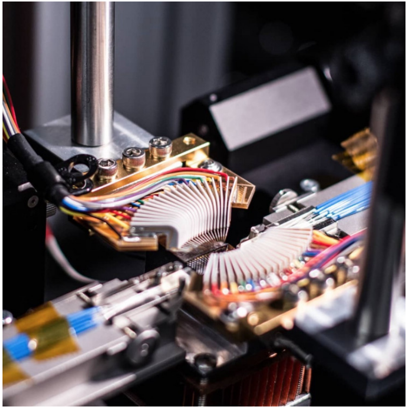
D-Wave quantum computer.
Font: cloud.dwavesys.com/leap/signup

*weaker qubits

Xanadu

Quantum hardware

- Photonic QPUs
- Room temperature
- Continuous-variable
- Cloud access for partners and clients
  - 8, 12, and 24 qumodes*



Xanadu quantum processor.
Font: @xanadu.ai

*basic computation unit of the continuous-variable model.

Xanadu

Software

- Strawberry fields - Python
  - For Xanadu hardware
  - Simulator
- PennyLane - Python
  - Hardware independent
  - Quantum machine learning
  - Optimization
  - Quantum chemistry

```
import strawberryfields as sf
from strawberryfields import ops
from strawberryfields.utils import random_interferometer

# generate a unitary
U = random_interferometer(4)

prog = sf.Program(8)

# construct the quantum photonic program
with prog.context as q:
  ops.S2gate(1.0) | (q[0], q[4])
  ops.S2gate(1.0) | (q[1], q[5])
  ops.S2gate(1.0) | (q[3], q[7])

  ops.Interferometer(U) | q[:4]
  ops.Interferometer(U) | q[4:]

  ops.MeasureFock() | q

# execute the program on hardware
eng = sf.RemoteEngine(”X8”)
results = eng.run(prog, shots=10000)
print(results.samples)
```

Example of Python with Strawberry Fields.
Font: xanadu.ai/cloud-platform

Xanadu

Software

- Strawberry fields - Python
  - For Xanadu hardware
  - Simulator
- PennyLane - Python
  - Hardware independent
  - Quantum machine learning
  - Optimization
  - Quantum chemistry

```python
import pennylane as qml
from pennylane import numpy as np

# create a quantum device
dev1 = qml.device('default.qubit', wires=1)

@qml.qnode(dev1)
def circuit(phi1, phi2):
  # a quantum node
  qml.RX(phi1, wires=0)
  qml.RY(phi2, wires=0)
  return qml.expval(qml.PauliZ(0))

def cost(x, y):
  # classical processing
  return np.sin(np.abs(circuit(x, y))) - 1

# calculate the gradient
dcost = qml.grad(cost, argnum=[0, 1])
```

Example of Python with PennyLane.
Font: pennylane.ai

Microsoft Quantum Development Kit

Software

- Q# - Quantum DSL
  - Python, C#, and F#
  - Quantum libraries
  - Simulator
  - Resource estimator
- Quantum Katas
  self-paced exercises

```
namespace Microsoft.Quantum.Samples.Teleportation {
    open Microsoft.Quantum.Intrinsic;
    open Microsoft.Quantum.Canon;
    open Microsoft.Quantum.Measurement;

    operation Teleport (msg : Qubit, target : Qubit) : Unit {
        using (register = Qubit()) {
            H(register);
            CNOT(register, target);
            CNOT(msg, register);
            H(msg);
            if (MResetZ(msg) == One) { Z(target); }
            if (IsResultOne(MResetZ(register))) { X(target); }
        }
    }

}
```

Q# example.

Software

- Ket Quantum Programming
  - Ket - Python-embedded quantum language
  - Libket - C++ library
  - Ket Bitwise Simulator
- Cirq
- ProjectQ
- Silq
- Quipper

Thank you

For your attention

III Workshop de Computação Quântica - UFSC

Quantum computation for beginners.

Questions?

slido.com #70967