IV Workshop de Computação Quântica - UFSC
QAOA Algorithm

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Variational Quantum Algorithms

Variational Quantum Algorithms (VQAs) are a class of hybrid quantum-classical algorithms that employ a classical optimizer to train a parametrized quantum circuit and provide a general framework that can be used to solve a wide array of problems.

VQAs emerged as the leading strategy to obtain quantum advantage on NISQ devices.

Variational Quantum Algorithms

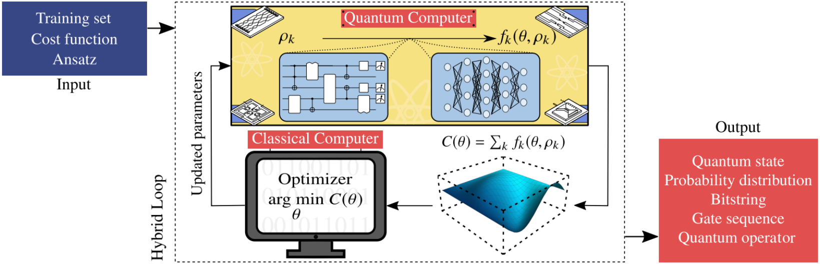


Figure: Schematic diagram of a Variational Quantum Algorithm (VQA).

QAOA Algorithm

- **Quantum Approximate Optimization Algorithm:** in its first publication by Farhi et al [FGG14] the QAOA was an approximate optimization algorithm, whose behaviour was based in the alternated application of a Hamiltonian based in a cost function and a Mixing Hamiltonian.
- **Quantum Alternating Operator Ansatz:** the QAOA framework was later expanded by Hadfield et al [HWO$^+$19] so that it would allow families of more general operators.

QAOA Circuit

$$H_P \Rightarrow e^{-i\gamma H_P} = -\boxed{U_P(\gamma)}- \qquad H_M \Rightarrow e^{-i\beta H_M} = -\boxed{U_M(\beta)}-$$

$$QAOA_p(\beta, \gamma) = U_M(\beta_p)U_P(\gamma_p) \dots U_M(\beta_1)U_P(\gamma_1)$$

$$|\beta, \gamma\rangle = QAOA_p(\beta, \gamma) |s\rangle$$

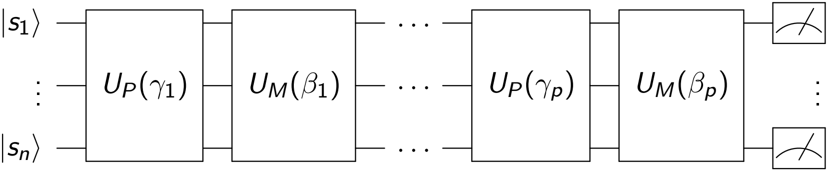
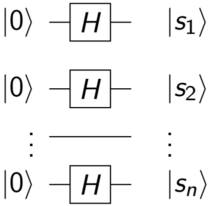


Figure: QAOA quantum circuit

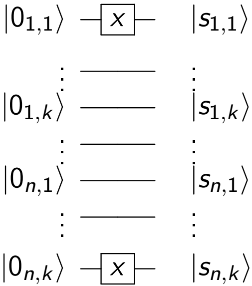
Initial State

Design criteria: The initial state must be feasible and *trivial*, $O(1)$ or $O(\log n)$, to implement

$|+ \ldots +\rangle$-state $O(1)$

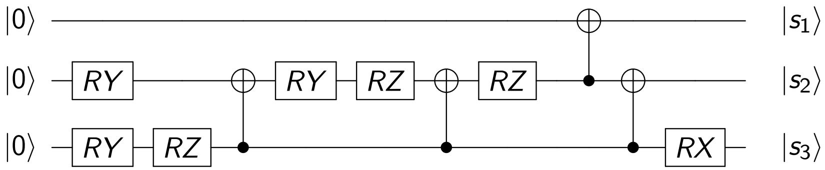


Feasible State $O(1)$



Initial State

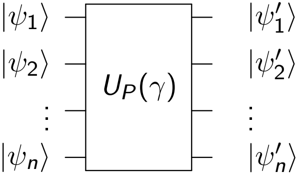
W-state $O(\log n)$



3-Qubit W-state: $|100\rangle + |010\rangle + |001\rangle$

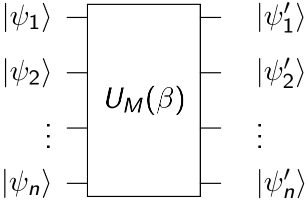
Phase separation operator

Design criteria: We require the family of phase separation operators $U_P(\gamma)$ to be diagonal in the computational basis. Most of the time the operator will take the form $U_P(\gamma) = e^{-i\gamma H_P}$, where H_P is the Problem Hamiltonian of the classic objective function f.



Mixing Unitaries (Mixers)

Design criteria: The mixing operators must preserve feasible space and explore the feasible space.



Mixers

- *X-Mixer*: $H_M = \sum_j X_j$

$$U_M(\beta) = e^{-i\beta H_M} = \prod_j e^{-i\beta X_j}$$

- *XY-Mixer*: $H_M = \sum_{i,j} X_i X_j + Y_i Y_j$

$$U_M(\beta) = e^{-i\beta H_M} = \prod_{i,j} e^{-i\beta X_i X_j + Y_i Y_j}$$

QAOA Classical part

The expectation value $\langle x| H_P |x\rangle$ of the problem Hamiltonian is estimated by taking the average of the states obtained by the measurements.

$$\langle x| H_P |x\rangle \equiv f(x)$$

The expectation value is then optimized using a classical optimization method over the $2p$ parameters γ and β.

QAOA Classical part

- Off-the-shelf gradient-free [WHT16, MW19, AASG20]
- Off-the-shelf gradient-based [VGS+20, ZWC+20, MC21]
- Deep Reinforcement Learning [KSC+20]
- Metalearning [YWZ+21]

A recent study [FPCV+21] showed that some algorithms which have great performance with QAOA are Adam, Conjugate Gradient, COBYLA, Nelder-Mead, Powell and SPSA.

Practical Example - MaxCut

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